Fault-tolerant Sliding Mode Control for Uncertain Over-actuated Systems

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Abstract: This paper presents the sliding mode control design based on nonlinear sliding surface that ensures high performance step response of uncertain over-actuated system. High performance sliding mode control is designed for the system with predefined virtual control inputs having order less than the order of the system. Total control effort is then distributed among the actuators using pseudo-inverse control allocation technique. To make the system fault-tolerant (actuator saturation tolerant), the control allocation is updated online in a predefined way.

Keywords: Sliding mode control, over-actuated system, control allocation, fault-tolerant system, nonlinear sliding surface.

1. INTRODUCTION

Many safety critical modern control applications such as flight control, marine vessel control, nuclear fusion control use redundant actuators inducing the same effect on the plant dynamics. The system that uses more control effectors (actuators) than axes (states) to control may be referred to as over-actuated or redundant input system. In such situations, the mixing or allocation of effectors is necessary to achieve the desired objective. Control allocation problem is the distribution of a control demand among an available set of actuators. Control allocation is done in such a way that it resolves the actuator constraints like the saturation in position or rate and utilizes the maximum available control power, refer (Zhang and Jiang, 2008).

Various linear control allocation techniques such as explicit ganging, pseudo inverse, daisy chaining, direct allocation, error and control minimization and mixed allocation problems are discussed in (Oppenheimer et al., 2006) and references therein. Liao et al. (2007) have proposed an optimal control allocation method for a general class of overactuated nonlinear systems, which is based on Lyapunov design approach with finite-time convergence to optimality and for the NMP system with unstable internal dynamics in (Benosman et al., 2009). The dynamic control allocation problem using the generalized (pseudo) inverse technique has been solved in (Harkegard, 2004) and (Harkegard and Glad, 2005). In (Kishore, 2008), control allocation using constrained optimal pseudo inverse solution and disturbance observer is used for uncertain systems.

Alwi and Edwards (2006) have discussed fault tolerant sliding mode control scheme via weighted pseudo inverse allocation. Hamayun et al. (2010) have used integral sliding mode control to start the sliding mode from the beginning.

To improve the step response of second order system in terms of settling time and overshoot specifications, an idea of using CNF controller is proposed in (Lin et al., 1998) and further these results are extended to the higher order and multiple input systems by Turner et al. in (Turner et al., 2000). In (Chen et al., 2003), CNF controller consists of a linear and a nonlinear feedback law which is used for the continuous and the discrete-time linear systems with actuator saturation. The linear feedback part is designed to yield closed-loop system with small damping ratio for a quick response for the desired command input levels and the nonlinear feedback law is used to increase the damping ratio of the closed-loop system as the system output approaches the target reference to reduce the overshoot caused by the linear part.

Sliding mode control (SMC) has been a topic of great interest for researchers because of insensitivity of the system to the matched disturbance during a sliding motion (Edwards and Spurgeon, 1998). Comprehensive surveys on SMC can be found in literature (e.g. Utkin, 1977; Decarlo et al., 1988; Hung et al., 1993). To achieve improved transient response specifications the nonlinear sliding surface design proposed in (Bandyopadhyay and Fulwani, 2009), ref. also (Bandyopadhyay et al., 2009). Various techniques in sliding mode control for fault-tolerance can be found in (Alwi et al., 2011, and references therein).

In this paper, sliding mode control with nonlinear sliding surface for uncertain over-actuated system is designed to achieve high performance transient response specifications. The control allocation problem is separated from the controller design to avoid controller reconfiguration in event of actuator saturation. The control space dimension of over-actuated system is reduced to virtual control space. Sliding mode control is designed for under-actuated system model with virtual input and the total control effort is distributed among all the actuators using control allocation unit.
2. PROBLEM FORMULATION

Consider the multivariable linear time invariant system
\[ \dot{x}(t) = Ax(t) + B\delta(t) + B_d(t) \]
\[ y(t) = Cx(t) \]
where \( x(t) \in \mathbb{R}^n, \delta(t) \in \mathbb{R}^m, y(t) \in \mathbb{R}^p \) and \( d(t) \in \mathbb{R} \) are state, input, output and disturbance vectors of the system respectively. Dimensions are assumed as \( p < n < m \).

\( \delta \) and \( \delta_{\text{min}} \) are upper and lower bounds on position of \( \delta \), while \( \delta_{\text{max}} \) and \( \delta_{\text{min}} \) are maximum and minimum control rates defined for \( \delta \). However, the control rate constraints are converted into position constraints in digital control algorithm and can be combined to the position limits. The matrix \( B \) is factored into having full column rank and row rank, respectively.

Define the virtual input
\[ u(t) := E\delta(t) \]
The original system (1) with virtual input \( u(t) \) as defined in (4) is appeared as
\[ \dot{x}(t) = Ax(t) + B_1u(t) + B_d(t) \]
The control space dimension of the system is reduced from \( \mathbb{R}^m \) to \( \mathbb{R}^p \) and the virtual input \( u(t) \) can be treated as total control effort which should be distributed further among \( m \) actuators via control allocation. Advantage of reduction in control space dimension is that the controller design is separated from the allocation problem. First section of the control problem is the sliding mode controller design which ensures matched disturbance rejection along with the fast and oscillation free tracking performance for constant reference. The second section is the control allocation problem that resolves the actuator constraints and distribute the total control effort among the control effectors (Ref. Figure 1).

![Fig. 1. Control scheme for over-actuated system.](image)

Transforming the system (5) into regular form using the transformation
\[ z(t) := T_\tau x(t) \]
such that \( B_{\text{reg}} := T_\tau B_1 = \begin{bmatrix} 0 & B_2^T \end{bmatrix}^T, \bar{d}(t) := T_\tau B_d(t) = \begin{bmatrix} 0 & \bar{d}_T(t) \end{bmatrix}^T, \) and \( A_{\text{reg}} := T_\tau A T_\tau^{-1}. \)

Then the system in regular form is represented as
\[ \dot{z}(t) = A_{\text{reg}}z(t) + B_{\text{reg}}u(t) + \bar{d}(t) \]
Define
\[ z(t) := \begin{bmatrix} z_1(t) \ z_2(t) \end{bmatrix}^T, C_{\text{reg}} := C(T_\tau)^{-1} \]
\[ A_{\text{reg}} := \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \]
Then the partitioned system in regular form is represented as,
\[ \dot{z}_1(t) = A_{11}z_1(t) + A_{12}z_2(t) \]
\[ \dot{z}_2(t) = A_{21}z_1(t) + A_{22}z_2(t) + B_2u(t) + \bar{d}_2(t) \]
\[ y(t) = C_{\text{reg}}z(t) \]

**Assumptions**

A.1 \((A, B_1)\) pair is stabilizable and \((A, C)\) pair is observable.
A.2 There exists the matrix \( F \) such that \((A_{11} - A_{12}F)\) is Hurwitz.
A.3 There exists the symmetric positive definite matrix \( P \) that satisfies the following Lyapunov equation for some positive definite matrix \( W \).
\[ (A_{11} - A_{12}F)^T P + P(A_{11} - A_{12}F) = -W \]

3. CONTROL ALLOCATION

The primary objective of the control allocation is to distribute the overall control effort among the actuators by satisfying the actuator constraints. The virtual control law designed with reduced control dimension is to be spread across the control effectors in a constrained manner. Constrained linear control allocation problem for the system (1) is formulated as
\[ E\delta(t) = u(t) \]
with combined position and rate constraints of the actuators
\[ \delta_{\text{min}} \leq \delta \leq \delta_{\text{max}} \]
where
\[ \delta = \max(\delta_{\text{min}}, \delta - \Delta\hat{\delta}_{\text{max}}) \]
\[ \hat{\delta} = \min(\delta_{\text{max}}, \delta + \Delta\hat{\delta}_{\text{min}}) \]

**Remark 1.** In digital implementation of the algorithm the rate limits can be easily converted into position limit during sampling period \( \Delta t \) (see Oppenheimer et al., 2006).

If there exists no solution to the (14), control allocation problem may be solved by minimization of \( \|E\delta(t) - u(t)\| \). When there are many solutions available, a method to pick optimum one is necessary. Thus the constrained optimization problem becomes determination of \( \delta \) that satisfies the constraints (15) at every instant of time and minimizing the magnitude of actuator position i.e. \( \min_{\delta} \delta^TW_\delta \delta \). Such a problem has weighted pseudo inverse solution if actuator constraints are relaxed (Harkegard and Glad, 2005).

\[ \delta = W_\delta^{-1}E^T EW_\delta^{-1}E^T u \]

where,
\[ W_\delta = \text{diag}(w_{\delta 1}, \ldots, w_{\delta p}) > 0 \]
\( W_\delta \) can be choosen as identity matrix, which gives the MoorePenrose pseudo-inverse refer (Shtessel et al., 2002). However when at least one actuator exceeds the solution (17) may not be admissible. So in case of actuator saturation it is necessary to find a new solution. Thus online adaptive control allocation that updates the weight matrix \( W_\delta \) is needed.
3.1 Adaptive control allocation

An element of the weight matrix $W_δ$ is updated in the event of fault in corresponding actuator. The updation is based on the switching rules described in the section 3.2. Let $i^{th}$ diagonal element of $W_δ$ is perturbed from $w_{si}$ to $k_{si}^2w_{si}$, $k_{si} > 0$ then $W_δ$ is updated as

$$W_δ = diag(w_{s1} \cdots k_{s1}^2w_{s1} \cdots w_{sp})$$

This result can be generalized as

$$W_δ = \delta^2 \delta w_\delta$$

where

$$\delta w_\delta = diag(k_{s1} \cdots k_{sp})$$

The control input via allocation is given by

$$\hat{\delta} = \hat{W}_δ^{-1}E^T(E\hat{W}_δ^{-1}E)^{-1}u$$

3.2 Selection of $\delta w_\delta$

Weight of $i^{th}$ diagonal element of $W_δ$ is updated via appropriate change in $(k_{s1})^{th}$ element of $\delta w_\delta$ matrix, see (Buffington et al., 1999). For normal operation of the $i^{th}$ actuator, $k_{si} = 1$. If actuator saturates then

$$k_{si} = (1 + \varepsilon) \frac{\delta_i}{\delta}, \quad \text{if } \delta_i < \delta$$

$$k_{si} = (1 + \varepsilon) \frac{\delta_i}{\delta}, \quad \text{if } \delta_i > \delta$$

Where $\varepsilon$ is small positive tuning parameter that determines the rate of convergence of $\delta_i$ to region $S_δ$

4. SLIDING MODE CONTROL

4.1 Tracking problem

Consider the system (7) without disturbance.

$$\dot{z}(t) = A_{reg}z(t) + B_{reg}u(t)$$

For the tracking problem the desired trajectory needs to be generated consistently. As $(A_{reg}, B_{reg})$ pair is stabilizable, there exists the some control that generates the desired trajectory.

$$u_d(t) = -F_d(z(t) + Gr(t))$$

Where $r(t)$ is an exogenous command input and $G$ is a scalar that can be obtained as

$$G := \left[ C_{reg}(A_{reg} - B_{reg}F_d) \right]^{-1}B_{reg}\Psi(t)$$

Such a scalar $G$ exists because by assumptions $(A_{reg} - B_{reg}F_d)$ is stable and invertible and no invariant zeros at $s = 0$. Here $[\cdot]^\dagger$ represents the general inverse. And the desired state $z_d(t)$ is computed as

$$z_d(t) := -(A_{reg} - B_{reg}F_d)\Psi(t)$$

The dynamics of the system generating the desired states is given by

$$\dot{z}_d(t) = A_{reg}z_d(t)$$

Define mismatch in variables

$$\Delta z(t) := z(t) - z_d(t)$$

$$\Delta u(t) := u(t) - u_d(t)$$

$$\Rightarrow \begin{bmatrix} \Delta z_1(t) \\ \Delta z_2(t) \end{bmatrix} = \begin{bmatrix} z_1(t) - z_{d1}(t) \\ z_2(t) - z_{d2}(t) \end{bmatrix}$$

From (10)-(11) and definitions (29)-(30) the tracking system is formulated as

$$\Delta \dot{z}_1(t) = A_{11}\Delta z_1(t) + A_{12}\Delta z_2(t)$$

$$\Delta \dot{z}_2(t) = A_{21}\Delta z_1(t) + A_{22}\Delta z_2(t) + B_2\Delta u(t) + d_2(t)$$

The control for the mismatch dynamics (31a)-(31b) can now be designed such that $\|\Delta z(t)\rightarrow 0\|$ as $t \rightarrow \infty$

4.2 Sliding surface design

Let the sliding surface for the system in (31a) and (31b) be defined as

$$s(\Delta z(t)) := c^T(\Delta z(t)) := [c_1(t) \ I_m] \begin{bmatrix} \Delta z_1(t) \\ \Delta z_2(t) \end{bmatrix}$$

For stable sliding motion, virtual control $c_1$ is chosen such that $(A_{11} - A_{12}c_1)$ is Hurwitz. However, to achieve the high performance response in terms of step response specifications, $c_1$ can be designed as nonlinear function

$$c_1(t) := -F - \Psi(y(t))A_{12}^TP$$

Where $P$ is some positive definite symmetric matrix satisfying the equation (13) and $\Psi(y(t))$ is a digonal matrix of non-positive nonlinear functions which is used to change the damping ratio of reduced order subsystem as the output approaches the reference input. The nonlinear function is chosen such that its value changes from 0 to $-\beta$. This leads to the gradual change in $c_1(t)$ from the initial gain $F$ designed for small risetime to the $K_2$ to yield overdamped response as tracking error tends to zero. Possible choice of $\Psi(y(t))$ is as follows,

$$\Psi(y(t)) = \begin{bmatrix} \psi_1(t) \\ \psi_2(t) \\ \vdots \\ \psi_p(t) \end{bmatrix}$$

$$\psi_i(t) = -\beta_i \exp(-\alpha_i |y_i(t) - r_i(t)|), \quad i = 1, 2, \cdots, p$$

Where $\beta$ and $\alpha$ are tuning parameters. The value of $\beta$ contributes the change in controller gain whereas $\alpha$ determines the rate of change of $\Psi(y)$.

4.3 Stability during sliding mode

During sliding motion switching function is $s(\Delta z, t) = 0$, it follows that

$$\Delta \dot{z}_2(t) = -F\Delta z_1(t) + \Psi(y(t))A_{12}^TP\Delta z_1(t)$$

substituting in (31a), we get

$$\Delta \dot{z}_1(t) = (A_{11} - A_{12}F + A_{12}\Psi(y(t))A_{12}^TP)\Delta z_1(t)$$

Consider that $V(\Delta z_1(t)) = \Delta z_1(t)^TP\Delta z_1(t)$ be the candidate Lyapunov function for the subsystem (37). The time derivative of the $V(\Delta z_1(t))$ along the solution of the subsystem (37) is given by

$$\dot{V}(\Delta z_1(t)) = \Delta z_1(t)^TP\Delta z_1(t) + \Delta z_1(t)^TP\Delta z_1(t)$$

substituting (37) into (38) and simplify, we get

$$\dot{V}(\Delta z_1(t)) = \Delta z_1(t)^T\left[-W + 2PA_{12}\Psi(y(t))A_{12}^TP\right]\Delta z_1(t)$$

By definition $\Psi(y(t)) < 0$ and $W > 0$, Thus $\dot{V}(\Delta z_1(t)) < 0$. It follows that sliding motion is stable.
5. SLIDING MODE CONTROL AND ALLOCATION

The sliding mode control law is designed such that trajectory of the system from any initial condition should move to the sliding surface in finite time and force to remain on it.

**Proposition 2.** For the system (31), if $K > 0$ and the $i^{th}$ element of diagonal switching gain matrix $Q$ satisfies the condition

$$Q_i \geq |\tilde{d}_2(i)|$$

(40)

the control law

$$\dot{\delta}(t) = - (\bar{W}_s^{-1}E^T(\bar{W}_s^{-1}E^T)^{-1}B_2^{-1})^{-1}[\psi A_{reg} \Delta z(t)$$

$$- \hat{\psi}(y(t))A_{12}^T P \Delta z_1(t) + K_s + Q sgn(s)]$$

(41)

enforces the trajectory $\Delta z(t)$ of the system from any initial condition to reach the sliding surface $s(t)$ in (32) in a finite time and thereafter it remains thereon.

**Proof.** Consider that $V(s) = 0.5s^T s$ be the Lyapunov function for the sliding surface (32). Time derivative along the surface is given by

$$V(s) = s^T \dot{s}$$

$$= s^T [F \Delta \dot{z}_1(t) - \hat{\psi}(y(t))A_{12}^T P \Delta z_1(t)$$

$$- \hat{\psi}(y(t))A_{12}^T P \Delta z_1(t) + \Delta \dot{z}_2(t)]$$

$$= s^T [(F - \Psi(y(t))A_{12}^T P) \Delta z_1(t)$$

$$+ \Delta \dot{z}_2(t) - \hat{\psi}(y(t))A_{12}^T P \Delta z_1(t)$$

$$= s^T [c_1(A_{11} \Delta z_1(t) + A_{12} \Delta z_2(t))$$

$$+ A_{21} \Delta z_1(t) + A_{22} \Delta z_2(t)$$

$$+ B_2 u(t) + \dot{d}_2(t) - \hat{\psi}(y(t))A_{12}^T P \Delta z_1(t)]$$

$$= s^T [c_1 \Delta z_1(t) + A_{12} \Delta z_2(t))$$

$$+ A_{21} \Delta z_1(t) + A_{22} \Delta z_2(t)$$

$$+ B_2 u(t) + \dot{d}_2(t)]$$

(42)

Follow from (22) and (42)

$$V(s) = s^T [c \Delta z - \hat{\psi}(y(t))A_{12}^T P \Delta z_1(t)$$

$$+ B_2 (\bar{W}_s^{-1}E^T(\bar{W}_s^{-1}E^T)^{-1}B_2^{-1}) \delta(t)$$

$$+ \dot{d}_2(t)]$$

(43)

Substituting the control law (41) into (43) yield,

$$V(s) = s^T [-Ks - Q sgn(s)] + \dot{d}_2(t)]$$

$$\leq \begin{bmatrix} s_1 & \cdots & s_p \end{bmatrix} \begin{bmatrix} -Q sgn(s_1) \\ \vdots \\ -Q sgn(s_p) \end{bmatrix}$$

$$+ s_1 \tilde{d}_{21} + \cdots + s_p \tilde{d}_{2p}$$

(44)

Using condition (40), it is verified that for some $\eta_i > 0$, $i^{th}$ element of $V(s)$ satisfies the reachability condition

$$V_i(s) < -\eta_i |s_i|$$

(45)

This completes the proof.

**Remark 3.** Computation of derivative of nonlinear surface gain $\hat{c}_1(t)$ in control law (41) is possible using exact differentiator, refer (Levant, 1998).

**Remark 4.** $(\bar{W}_s^{-1}E^T(\bar{W}_s^{-1}E^T)^{-1}B_2^{-1})$ is updated online according to (23)-(24).

6. NUMERICAL SIMULATION

In this section, attitude control of satellite launch vehicle (SLV) as used in (Kishore, 2008) is given as an illustrative example to verify the effectiveness of the sliding mode control design method for over-actuated systems. SLV is three dimensional structure. The rigid body dynamics of SLV are constituted by forces and moments due to control thrust deflections, aerodynamic forces, attitude changes and slosh. The net forces can be resolved along three directions. The moment components along these axes are pitching, yawing and rolling moments. Pitch $(\dot{\theta})$, yaw $(\psi)$ and roll $(\phi)$ are the controlled variables. The input to the system are strapon thrusters $(\delta_1, \delta_2, \delta_3, \delta_4)$ and core thrusters $(\delta_5, \delta_6, \delta_7, \delta_8)$. Disturbances to the system are due to the difference in core thrusters $(d_1)$, strapon thrusters $(d_2)$ and deflections due to mounting misalignment along pitch, yaw and roll $(d_3, d_4, d_5)$.

The state space model of SLV is given by

$$\dot{x}(t) = Ax(t) + B\delta(t) + B_d d(t)$$

$$y(t) = C x(t)$$

(46)

(47)

where

$$x(t) = \begin{bmatrix} \dot{\theta} \ \dot{\psi} \ \dot{\phi} \ \dot{\psi} \end{bmatrix}^T$$

$$\delta(t) = \begin{bmatrix} \delta_1 \ \delta_2 \ \delta_3 \ \delta_4 \ \delta_5 \ \delta_6 \ \delta_7 \ \delta_8 \end{bmatrix}^T$$

$$d(t) = \begin{bmatrix} d_1 \ d_2 \ d_3 \ d_4 \ d_5 \ d_6 \ d_7 \ d_8 \end{bmatrix}^T$$

$$y(t) = \begin{bmatrix} \theta \ \psi \ \phi \end{bmatrix}^T$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1.7168 & -1.7159 & -1.7163 & 1.7164 & 0.45 & -0.0009 & -0.4480 & 0 \\ 0.0008 & 0.2104 & 0.0003 & 0.0003 & 0.0003 & 0.0003 & 0.0003 & 0.0003 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_d = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Combined position and rate constraints for $\delta$ are

$$\dot{\delta}(t) = - \begin{bmatrix} 5 & 5 & 5 & 5 & 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}^T$$

$$\ddot{\delta}(t) = \begin{bmatrix} 5 & 5 & 5 & 5 & 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}^T$$

(48)

(49)

Factorize $B$ into the full column rank matrix $(B_1)$ and full row rank matrix $(E)$,

$$B_1 = \begin{bmatrix} 5.7508 & 0.0002 & -0.0015 \\ 0.0002 & 5.1730 & 0.0021 \\ -0.0046 & 0.217 & 14.1528 \end{bmatrix}$$

$$E = \begin{bmatrix} 0.2985 & -0.2985 & 0.2985 & 0.2985 & 0.2985 & 0.2985 & 0.2985 & 0.2985 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2985 & -0.2985 & 0.2985 & 0.2985 & 0.2985 & 0.2985 & 0.2985 & 0.2985 \\ -0.1618 & 0.1618 & -0.1618 & 0.1618 & 0.1618 & 0.1618 & 0.1618 & 0.1618 \end{bmatrix}$$

Transforming the reduced space control dimension system $(A, B_1)$ into regular form using transformation matrix
The regular form model is obtained as

\[
A_{reg} = \begin{bmatrix}
-0.0034 & 0 & 0 & 0.0034 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0 & 0 & 0.0000 & 0 & 0 & 0 \\
0.0000 & 0 & 0 & 0.0000 & 0 & 0 & 0 \\
-0.0000 & 0 & 0 & 0.0000 & 0 & 0.0000 & 0.0000 \\
0.0000 & 0 & 0.0000 & 0 & 0 & 0 & 0 \\
0.0000 & 0.0000 & 0 & 0 & 0 & 0 & 0 \\
0.0000 & 0.0000 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
B_{2} = \begin{bmatrix}
5.7508 & 0.0004 & 0.0129 \\
0.0000 & 5.1730 & 0.0614 \\
0.0000 & 0.0000 & 14.1526
\end{bmatrix}
\]

\[
C_{reg} = \begin{bmatrix}
1.0000 & 0 & 0.0014 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
-0.0034 & 0 & 1.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 \\
0.0000 & 0 & 0.0000 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

To design the nonlinear sliding surface initially choosing the state feedback matrix

\[
F = \begin{bmatrix}
1.5024 & 2.9988 & 0.0103 \\
-0.0177 & -0.0640 & 3.0000 \\
-2.9987 & 1.5025 & -0.0126
\end{bmatrix}
\]

for the closed loop damping factor of 0.447 of dominant poles of subsystem.

The nonlinear feedback controller parameters are chosen as \(\beta_1 = \beta_2 = \beta_3 = 0.93\), and

\[
P = \begin{bmatrix}
53.1251 & 0 & 0 \\
0 & 53.1251 & 0 \\
0 & 0 & 71.8192
\end{bmatrix}
\]

and tuning parameter \(\alpha_1 = \alpha_2 = \alpha_3 = 0.4\). The composite controller design results into the state feedback gain,

\[
K_2 = \begin{bmatrix}
50.8598 & 3.0411 & 0.2320 \\
0.1614 & 0.2336 & -67.7317 \\
3.0418 & -50.8596 & -0.2839
\end{bmatrix}
\]

at the steady state. The disturbance vector considered for this illustration is \(d(t) = \sin(5t) \times [1 \ 1 \ 0.1 \ 0.1 \ 0.1]^T\).

The Sliding mode control parameters are chosen as \(K = diag(3, 3, 3)\) and \(Q = diag(1.5, 1.5, 1.5)\).

In fig. (2), (3), (4), pitch, yaw and roll responses due to control law with linear surface gain \(F\) and \(K_2\) designed for underdamped and overdamped responses are compared with the response of the system with nonlinear switching surface. Total sliding mode control inputs and actual control surfaces are shown in fig. (5) and (6) respectively.

It can be seen from fig. (7) that actuators \(\delta_1 \) to \(\delta_4\) are initially driven into saturation, so their weights are updated to 0.25, 0.25, 0.25 and 6.25 while remaining actuators are at their nominal weights.

7. CONCLUSION

It is possible to design the sliding mode control for overactuated system that ensures the performance and robustness of the system against the matched disturbance. This can be achieved by sliding mode control based on nonlinear sliding surface. The sliding mode control is designed with predefined virtual control inputs having order less than the order of the system and further total control effort is distributed among the actuators using pseudo-inverse control allocation technique. The control allocation is updated online in the event of actuator faults.

REFERENCES


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**Fig. 4.** Roll response of the system.

**Fig. 5.** Total Control effort (Virtual Control inputs)

**Fig. 6.** Actuator Responses $\delta_{1}(t)$ to $\delta_{8}(t)$.

**Fig. 7.** Switching in Control allocation gains