Implementation of High Performance Nonlinear Feedback Control on Magnetic Levitation System

Deepti Khimani ∗ Sanket Karnik ∗∗ Machhindranath Patil ∗∗∗

∗ V.E.S. Institute of Technology, Mumbai, India.
(e-mail: deepti.khimani@ves.ac.in)
∗∗ L&T Hydrocarbon Engineering, Navi Mumbai, India.
(e-mail: sanket.karnik@ves.ac.in)
∗∗∗ V.E.S. Institute of Technology, Mumbai, India.
(e-mail: machhindra.patil@ves.ac.in)

Abstract: This paper presents the design of nonlinear state feedback that ensures high performance step response of magnetic levitation (maglev) system. The controller is designed for a maglev system in such a way that it exhibits the high speed response without exhibiting the overshoot. This is achieved by keeping the damping ratio initially small and as the trajectory approaches to the command value, damping ratio is made high to avoid the overshoot in response. The designed controller is implemented on the experimental setup Maglev 730 system developed by ECP systems and results of the experiment are compared with the simulation results.

Keywords: Magnetic levitation system, composite nonlinear feedback, output regulation, state feedback design, linear matrix inequality.

1. INTRODUCTION

Friction between moving surfaces can be reduced substantially using the magnetic levitation principle. Many industrial applications have been found in literature to have used a magnetic levitation in order to increase the overall efficiency of the system. For instance, Kaplan and Regev (1976) utilized magnetic levitation principle for high-speed train suspension. Design of magnetic bearings can be found in (Dussaux, 1990), supraconductor rotor suspension of gyroscopes in (Bencze et al., 1996) and wind turbines has been reported in (Hu, 2008; Kumbernuss et al., 2012). Magnetic levitation principle has been discussed in (Kaloust et al., 2004) for launch assist in space missions.

Usually, controllers for such a system are designed primarily to maintain the levitated object at a desired height and the performance during positioning that otherwise can be hindered by inherent nonlinearities. The design methods for linear and nonlinear state feedback controllers have been found in (Barie and Chiasson, 1996) and (El Hajjaji and Ouladsine, 2001), which are based on feedback linearization (Isidori, 2013). For the robustness against the parametric uncertainties, adaptive robust controller design has been addressed in Yang and Tateishi (2001). This method essentially involves a two-step control action. First, the levitated object is positioned using PI controller and second, adaptive robust feedback controller attenuates the effect of parametric uncertainties. Design of a robust 2-DOF (degree-of-freedom) controller using quantitative feedback technique and a nonlinear controller for magnetic levitation has been presented in Nataraj and Patil (2008) and Nataraj and Patil (2010), respectively.

The maglev system dynamics can be represented by a second order linear ordinary differential equation. If the motion of levitating object is considered to be frictionless then the system can be modeled as undamped, which exhibits an oscillatory response to the desired position. However, the laboratory setup on which experimentation has been done confines the motion of the levitating magnet along the glass rod. This provides a small amount of friction, though negligible. Therefore, the open loop response of the system becomes underdamped that results in decaying oscillations before it settles to the desired position.

In either case, the response exhibits overshoots. Therefore, it is essential to design the controller that does not exhibit overshoots. A traditional approach of the state feedback via pole placement can be employed easily to place poles of the system for overdamped response that does not exhibit overshoots (Belanger, 1995). However, rise-time of such response is large. To achieve the response with small rise-time without exhibiting the overshoots design of composite nonlinear feedback (CNF) controller is addressed in (Chen et al., 2003) for continuous-time systems and in (Venkataramanan et al., 2003) for discrete-time systems.

In this paper, we have designed and implemented the CNF controller for the Maglev Model 730 laboratory test bench developed by educational control products ECP (2016), which is an open loop underdamped system. An idea of CNF is utilized to achieve the desired position of levitating magnet with relatively small rise-time and
without exhibiting the overshoots. This can be achieved by keeping the damping ratio initially small. As the magnet approaches to the desired position, damping ratio is made high using nonlinear feedback control to avoid the overshoot in response. The results obtained experimentally are quite satisfactory when compared them with simulation results.

2. MODELING OF MAGNETIC LEVITATION SYSTEM

An experimental setup of ECP Model 730 Maglev system is shown in Fig.1. The experimental setup consists of two coils mounted on top and bottom that can be energized separately to levitate one or two magnets along a glass rod. This can be configured as single-input single-output (SISO) or multivariable system. In this paper, control of magnet position using lower coil is considered. A lower coil is energized to produce a repulsive magnetic force to levitate the magnet at the desired position. The resultant force on the levitated magnet position using lower coil is considered. A lower coil is energized to produce a repulsive magnetic force to levitate the magnet at the desired position.

The free body diagram of ECP Model 730 Maglev systems is shown in Fig.2. The resultant force on the levitated magnet at \( y(t) \) is given by,

\[
m\ddot{y}(t) + c\dot{y}(t) = F_u(t) - mg \tag{1}
\]

For this setup, the actuator response to input current \( i \) is nonlinear and is given by

\[
F_u(t) = \frac{i(t)}{a(b + y(t))^N} \tag{2}
\]

Where \( N, a \) and \( b \) are constants which can be determined by numerical approximation from the characteristics.

Thus, from (1) and (2), we get

\[
m\ddot{y}(t) + c\dot{y}(t) = \frac{i(t)}{a(b + y(t))^N} - mg \tag{3}
\]

Define the state variables, \( x_1 := y \) and \( x_2 := \dot{y} \) and let \( u \) be the input in terms of the DAC counts equivalent to the coil current \( i(t) \). Therefore, the system (3) can be represented as

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{c}{m}x_2 + \frac{u}{ma(b + x_1)^N} - g
\end{align*} \tag{4}
\]

The linear model of the system can be obtained by Jacobian linearization at the desired operating point \((x_{1o}, x_{2o}, u_0)\) on the trajectory of (4) as,

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix} =
\begin{pmatrix}
0 & 1 \\
\frac{c}{m} & -\frac{k'}{m} - \frac{1}{m} - \frac{a}{ma(b + x_1)^N}
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} +
\begin{pmatrix}
0 \\
\frac{k'}{m}
\end{pmatrix} u \tag{5}
\]

where,

\[
k' := \frac{Nu_u}{a(b + x_{1o})^{N+1}} \text{ and } k'' := \frac{1}{a(b + x_{1o})^N}
\]

As equilibrium point \((x_{1o}, x_{2o})\) satisfies (5), dynamics of the system can be represented in terms of error when it is perturbed. Define \( x_1^* := x_1 - x_{1o}, x_2^* := x_2 - x_{2o} \) and \( u^* := u - u_0 \). So the error dynamics for perturbed system is given by

\[
\begin{pmatrix}
\dot{x}_1^* \\
\dot{x}_2^*
\end{pmatrix} =
\begin{pmatrix}
0 & 1 \\
\frac{c}{m} & -\frac{k'}{m} - \frac{1}{m} - \frac{a}{ma(b + x_{1o})^N}
\end{pmatrix}
\begin{pmatrix}
x_1^* \\
x_2^*
\end{pmatrix} +
\begin{pmatrix}
0 \\
\frac{k'}{m}
\end{pmatrix} u^* \tag{6}
\]

Remark 1. As \( k' > 0 \) and if we consider no air resistance and no friction between the glass rod and magnet i.e. \( c = 0 \). This implies poles (eigenvalues) of the system at \( \pm j\sqrt{K/m} \) and thus the response will be oscillatory about equilibrium. However, practically there exist a finite amount of resistance that can be modeled as viscous friction, though very small. This results in underdamped transient response.

2.1 Sensor Calibration

The sensor employed in the system is optical-sensor consists of LASER source/detector pair with nonlinear output characteristics. As suggested in ECP (2016), a typical calibrated output of the sensor based on optical design and geometry of the laser/detector and magnet is given by

\[
x_{1\text{cal}} = \frac{e_1}{x_{1\text{raw}}} + \frac{f_1}{\sqrt{x_{1\text{raw}}}} + g_1 + h_1 x_{1\text{raw}} \tag{7}
\]

where \( x_{1\text{cal}} \) is the calibrated output of the sensor and \( y_{1\text{raw}} \) is the raw sensor output. The coefficients \( e_1, f_1, g_1 \) and \( h_1 \) can be found by regression based curve fitting method. The values of the constants in terms of DAC counts are shown in Table.1.

2.2 System Identification

The mass of the magnet to be levitated and gravitational force are known quantities. However, to compute \( k' \) and \( k'' \) parameters \( a, b \) and \( N \) is required. For the experimental setup, the values recommended by manufacturer as given in reference manual are \( b = 6.2 \) and \( 3 < N < 4.5 \).
Therefore, the constant \(a\) can be computed for the given control effort \(u \equiv i\) and corresponding magnet position \(y\). As the steady-state height of the magnet represents an equilibrium, i.e. \(F_n = mg\), therefore from (2) we can identify the parameter \(a\). For better accuracy, average of the computed \(a\) for various control values \(u\) and corresponding position \(y\) can be considered. The parameters for maglev system are summarized in Table 2.

Thus, the linear model identified for nominal (desired) calibrated output of \(x_{10} = 2\) cm and the nominal control of \(u_0 = 5000\) is given by,

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-395 & -4.5
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
1584
\end{bmatrix} u
\]  
(8)

Remark 2. The value of \(c/m = 4.5\) for the model in (6) is obtained by using the system identification toolbox of Matlab for the given nominal condition.

3. COMPOSITE NONLINEAR FEEDBACK CONTROL

The composite nonlinear feedback control (Turner et al., 2000; Chen et al., 2003) has attracted the attention of researchers for its ability to provide faster system response almost without overshoots. This motivates us to design such controller that positions the magnet quickly without exhibiting the overshoots. In this section, the composite nonlinear feedback control design method is discussed. Consider a second order linear time invariant system,

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]  
(9)

Assume that, \((A, B)\) is controllable and system has no invariant zero at \(s = 0\) in \(s\)-plane.

The composite nonlinear feedback control for (9) is given by

\[
u = u_L + u_N
\]  
(10)

Where, \(u_L\) is linear control law designed to obtain initial underdamped response and \(u_N\) is the nonlinear feedback control that contributes to the linear control law so that damping of the system increases gradually. As the response reaches the desired (reference) input, the control law \(u\) makes the system overdamped that does not exhibit the overshoots.

3.1 Linear Control Law

Suppose that \(F\) be the state feedback gain that can be designed via pole placement such that \((A+BF)\) has stable complex conjugate eigenvalues to get the desired initial underdamped response (refer Franklin et al., 2015). Then for the output regulation, typically the linear control law is given by,

\[
u_L = Fx + Gr
\]  
(11)

where \(r\) is reference input and \(G\) is feedforward gain to eliminate the steady state error given by,

\[
G = -\left( C(A+BF)^{-1}B \right)^{-1}
\]  
(12)

Note that, there exist the feedback matrix \(F\) and gain \(G\) as the system is assumed to be controllable and no invariant zeros of the system are at origin.

As \((A+BF)\) is stable, there exists a positive definite symmetric matrices \(P\) and \(Q\) that satisfies the Lyapunove equation,

\[
(A+BF)^\top P + P(A+BF) = -Q,
\]  
(13)

3.2 Nonlinear Control Law

The purpose of adding the nonlinear control law to the linear control law is to slow down the response by increasing the damping ratio gradually as the tracking error \(\|r - y\| \to 0\). In other words, the state feedback gain is made to change gradually from \(F\) and finally becomes \(K\) such that \((A+BK)\) has stable real eigenvalues that exhibit overdamped response. Typically, the nonlinear control law is given by,

\[
u_N = \rho(r, y)B^\top P(x - x_d)
\]  
(14)

where, \(x_d\) is desired state vector given by,

\[
x_d = -(A+BF)^{-1}BGr
\]  
(15)

and \(\rho(r, y)\) is nonpositive function which will change its value from \(-\beta\) to 0 as error approaches the 0. Possible choice of such function is,

\[
\rho(r, y) = -\beta \frac{\exp(-|r(0) - y(0)|) - \exp(-|r - y|)}{\exp(-|r(0) - y(0)|)}
\]  
(16)

Clearly, \(r = r(0)\) and \(y = y(0)\) gives initially \(\rho(r, y) = 0\) and finally when error \(|r - y|\) is zero, we get \(\rho(r, y) = -\beta\).

Thus, when the tracking error \(|r - y|\) is 0, then effective feedback gain will be

\[
K = F - \beta B^\top P
\]  
(17)

3.3 Selection of \(\beta\) and \(P\) via linear matrix inequality

The nonlinear nonpositive function \(\rho(r, y)\) as defined in (16) is an important function that achieves the high speed response without experiencing the overshoot. The value of \(\rho(r, y)\) chosen changes from 0 to \(-\beta\) that makes the initial gain \(F\) designed for small rise-time to the gain \(K\) to yield overdamped response as tracking error \(|r - y|\) \(\to 0\). It follows that \(K = F - \beta B^\top P\).

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**Table 1. Sensor calibration data**

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_1)</td>
<td>115720000</td>
</tr>
<tr>
<td>(f_1)</td>
<td>7208826</td>
</tr>
<tr>
<td>(g_1)</td>
<td>-30540</td>
</tr>
<tr>
<td>(b_1)</td>
<td>-0.2411</td>
</tr>
</tbody>
</table>

**Table 2. Parameters of maglev system**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>120</td>
<td>mass of magnet (gm)</td>
</tr>
<tr>
<td>(N)</td>
<td>4</td>
<td>by selection</td>
</tr>
<tr>
<td>(b)</td>
<td>6.2</td>
<td>provided by manufacturer</td>
</tr>
<tr>
<td>(g)</td>
<td>9.81</td>
<td>gravitational acc. (m/s²)</td>
</tr>
<tr>
<td>(a)</td>
<td>0.98</td>
<td>identified via curve fitting</td>
</tr>
</tbody>
</table>
\[ \beta \text{ can be computed by rearranging eq. (17). However, this } \beta \text{ may not yield the desired damping ratio for a matrix } P \text{ that satisfies the condition (13). So the positive definite matrix } P \text{ and scalar } \beta \text{ are searched such that they satisfy the stability condition and simultaneously yield the final gain } K \text{ close enough to the specified value. This can be done by formulating these equations into linear matrix inequality (Bandyopadhyay et al., 2009).} \]

\[ \begin{bmatrix} \mu I & E \\ E & \mu I \end{bmatrix} > 0 \]

\[ (A + BF)^\top P + P(A + BF) < 0 \]

\[ P > 0 \]

Where \( E := B^\top P + \beta^{-1}(K - F) \) and \( \mu \) is a small constant. Solving the LMI conditions (18) for \( P \) and \( \beta \) yields the expected result.

4. CONTROLLER DESIGN FOR MAGLEV SYSTEM

The experiment is performed for the levitation of an object equivalent to DAC 10000 counts. That is, for simulation and experimental step input is set to \( r = 10^4 \). For a fast initial response, we keep \( F = [0 0] \) as open loop response is quite faster as the damping factor is \( \xi = 0.1130 \). For the steady state correction the forward gain computed from (12) is \( G = 0.2494 \). Thus, the linear state feedback control law as in (11) is designed as

\[ u_L = Fx + Gr = 0.2494 r \quad (19) \]

The rise-time of magnetic levitation with control input (19) is 0.0573. However, the overshoot is 69.87%. See the simulation response to step input \( r \) in Fig.3 for \( \xi = 0.1130 \).

The final feedback gain \( K = [-0.1584 -0.0317] \) is selected for overdamped response that does not exhibit overshoot while feedforward gain computed for steady state correction is \( G = 0.4078 \). Therefore, designed linear feedback control for overdamped response is given by

\[ u_L = Kx + Gr = -0.1584x_1 - 0.0317x_2 + 0.4078 r \quad (20) \]

The response with feedback gain \( K \) is relatively slow as rise-time increases to 0.1471 sec. The simulation response results from (20) for \( \xi = 1 \) is shown in Fig.3.

In order to achieve the fast response without or with minimum overshoot, we design nonlinear feedback control as given in (10), (11) and (14). The parameters chosen for nonlinear control \( u_N \) are \( \beta = 0.2, \alpha = 0.0001 \) and

\[ P = \begin{bmatrix} 1 & 0.0005 \\ 0.0005 & 0.0001 \end{bmatrix} \]

and the nonlinear function chosen from (16) is

\[ \rho(r, y) = -0.2 \frac{\exp(-10^4) - \exp(-|10^4 - y|)}{\exp(-10^4)} \]

Also, for \( r = 10^4 \) and \( G = 0.2494 \), desired trajectory is computed as \( x_d = [10^4 0]^\top \) as given in (15). So nonlinear control part is given by,

\[ u_N = \rho(r, y) \left( 0.7920(x_1 - 10^4) + 0.1584x_2 \right) \quad (22) \]

Therefore, from (10), we get high performance nonlinear state feedback control as

\[ u = 0.2494 r + \rho(r, y) \left( 0.7920(x_1 - 10^4) + 0.1584x_2 \right) \quad (23) \]

Fig.3 shows the simulation responses to step input \( r = 10^4 \) with linear state feedback control and nonlinear feedback control. The comparison of transient performance in simulation with linear and nonlinear control is shown in Table 3.

![Fig. 3. Simulation of Levitation y(t) of magnet with linear and nonlinear feedback controllers.](image)

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Linear underdamped control</th>
<th>Linear overdamped control</th>
<th>Nonlinear control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain ( F )</td>
<td>( K )</td>
<td>( F \rightarrow K )</td>
<td></td>
</tr>
<tr>
<td>Damping factor ( \xi )</td>
<td>0.1130</td>
<td>1</td>
<td>0.486 \rightarrow 1</td>
</tr>
<tr>
<td>Overshoot (%)</td>
<td>69.87</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rise-time (sec.)</td>
<td>0.0573</td>
<td>0.1471</td>
<td>0.1020</td>
</tr>
<tr>
<td>Setting time (sec.)</td>
<td>1.6304</td>
<td>0.2624</td>
<td>0.1805</td>
</tr>
</tbody>
</table>

Table 4. Performance comparison of linear and nonlinear feedback controllers (practical).

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Linear underdamped control</th>
<th>Linear overdamped control</th>
<th>Nonlinear control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain ( F )</td>
<td>( K )</td>
<td>( F \rightarrow K )</td>
<td></td>
</tr>
<tr>
<td>Damping factor ( \xi )</td>
<td>0.1130</td>
<td>1</td>
<td>0.486 \rightarrow 1</td>
</tr>
<tr>
<td>Overshoot (%)</td>
<td>70.40</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rise-time (sec.)</td>
<td>0.0453</td>
<td>0.1415</td>
<td>0.0695</td>
</tr>
<tr>
<td>Setting time (sec.)</td>
<td>2.0114</td>
<td>1.5995</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Fig.4 shows magnetic levitation \( y(t) \) response to the control (19), which is obtained practically and compared it with the simulation response. Practical and simulation responses of levitation \( y(t) \) to the control (20), are shown in Fig.5.

High performance magnetic levitation responses obtained practically and by simulation are shown in Fig.6.

The performance comparison of levitation response using nonlinear control with the linear controls (19) and (20) is
shown Fig.7 and transient response specification comparison is shown in Table 4.

5. CONCLUSION

In this paper, nonlinear state feedback control is designed for magnetic levitation system to achieve the high performance transient response. The controller is designed in such a way that initially, the effective feedback gain set such that the transient response is underdamped with low rise-time. As the response approaches the final value (command input value), the feedback gain gradually changed using nonlinear feedback that gradually increases the damping factor of the closed loop system. Finally, when the response reaches to the final value, damping factor of the response becomes greater than or equal to 1. Therefore, it does not exhibit the overshoot. The designed controller is implemented on the experimental setup Maglev 730 developed by ECP systems. The results of the experiment are compared with simulation results and they are found to be satisfactory.

In order to eliminate the disturbances in the input channel of Maglev 730, sliding mode control based on the nonlinear sliding surface can be designed that yields high performance response.

REFERENCES


